

Possibility of a $J^{PC} = 3^{-+}$ state

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Motivated by the observation of several molecule candidates in the heavy quark sector, we discuss the possibility of a state with $J^{PC} = 3^{-+}$. In a one-boson-exchange model investigation for the S wave $C = + D^* \bar{D}_2^*$ states, one finds that the strongest attraction is in the case $J = 3$ and $I = 0$ for both π and σ exchanges. Numerical analysis indicates that this hadronic bound state may exist. If a state around the $D^* \bar{D}_2^*$ threshold (≈ 4472 MeV) in the channel $J/\psi\omega$ (P wave) is observed, the heavy quark spin symmetry implies that it is not a $c\bar{c}$ meson and the J^{PC} are very likely to be 3^{-+} .

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I. INTRODUCTION

Mesons with exotic properties play an important role in understanding the nature of strong interactions. The observation of the so called XYZ states in the heavy quark sector has triggered lots of discussions on their quark structures, decays, and formation mechanisms. It also motivates people to study new states beyond the quark model assignments.

The X(3872), first observed in the $J/\psi\pi^+\pi^-$ invariant mass distribution by Belle collaboration in 2003 [1], is the strangest heavy quark state. Even now, its angular momentum is not determined. Since its extreme closeness to the $D^0 \bar{D}^{*0}$ threshold, lots of discussions about its properties are based on the molecule assumption. However, it is very difficult to identify the X(3872) as a shallow bound state of $D^0 \bar{D}^{*0}$ since there are no explicitly exotic molecule properties.

A charged charmonium- or bottomonium-like meson labeled as Z is absolutely exotic because its number of quarks and antiquarks must be four or more. Such states include the $Z(4430)$ observed in the $\psi'\pi^\pm$ mass distribution [2], the $Z_1(4050)$ and $Z_2(4250)$ observed in the $\chi_{c1}\pi^\pm$ mass distribution [3], and the $Z_b(10610)$ and $Z_b(10650)$ in the mass spectra of the $\Upsilon(nS)\pi^\pm$ ($n=1,2,3$) and $\pi^\pm h_b(mP)$ ($m=1,2$) [4]. They are all observed by Belle Collaboration. Though BABAR has not confirmed them [5, 6], the existence signal of multiquark states is still exciting. Since $Z(4430)$ is around the $D^* D_1$ threshold, $Z_b(10610)$ is around the BB^* threshold, and $Z_b(10650)$ is around the $B^* B^*$ threshold, molecular models seem to be applicable to their structure investigations [7–13].

To identify a state as a molecule is an important issue in hadron studies. One should consider not only bound state problem of two hadrons, but also how to observe a molecular state in possible production processes. In Refs. [14–17], bound states of $\Sigma_c \bar{D}$ and $\Sigma_c \bar{D}^*$ were studied. Since the quantum numbers are the same as the nucleon but the masses are much higher, identifying them as multiquark baryons is rather apparent. To obtain a deeper understanding of the strong interaction, it is necessary to explore possible molecules with explicitly exotic quantum numbers.

Quark model gives us a constraint on the quantum numbers of a meson, namely, a meson with $J^{PC} = 0^{--}, 0^{+-}, 1^{-+}, 2^{+-}, 3^{-+}, \dots$ could not be a $q\bar{q}$ state, but it may be a multiquark state. So the study on such states may deepen our understanding of nature. If two $q\bar{q}$ mesons can form a molecule with such quantum numbers, one gets the simplest configuration. Next simpler configuration is the baryon-antibaryon case. A possible place to search for them is around hadron-hadron thresholds. There are some discussions on low spin heavy quark exotic states in Refs. [18, 19]. Here we would like to discuss the possibility of a higher spin state, $J^{PC} = 3^{-+}$. One will see that identification of it from strong decay is possible.

First, we check meson-antimeson systems that can form 3^{-+} states, where meson (antimeson) means that its quark structure is $c\bar{q}$ ($\bar{c}q$). The established mesons may be found in the Particle Data Book [20]. One checks various combinations and finds that the lowest S-wave system is $D^* \bar{D}_2^*$. The next S-wave one is $D_s^* D_{s2}^*$. Between these two thresholds, one needs D or G wave to combine other meson-antimeson pairs (see Fig. 1). Below the threshold of $D^* \bar{D}_2^*$, the orbital angular momentum is D , F , or G -wave. Above the $D_s^* D_{s2}^*$ threshold, a partial wave of P , F , or H is needed. Since the difference between these two thresholds is more than 200 MeV, one may neglect the channel coupling and choose the $D^* \bar{D}_2^*$ system to study.

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4684.2	$D_s^* D_{s2}^*$ [SDGI]
4647.4	$D_s^* D_{s1}^*$ [DG]
4571.9	$D_s^* D_{s1}'$ [DG]
4540.4	$D_s^* D_{s2}^*$ [DG]
4503.6	$D_s D_{s1}$ [DG]
4469.6	$D^* D_s^*$ [SDGI]

FIG. 1: Thresholds of $J^{PC} = 3^{-+}$ meson-antimeson systems between that of $D^* D_2^*$ and that of $D_s^* D_{s2}^*$. S , D , G , and I are orbital angular momenta.

Secondly, we check baryon-antibaryon systems. If one combines the established $cq\bar{q}$ baryons and their antibaryons, one finds that the lowest S-wave threshold is for $\Lambda_c(2880)$ and $\bar{\Lambda}_c$ (≈ 5168 MeV). Even for the lowest threshold of $\Lambda_c(2595)$ and $\bar{\Lambda}_c$ in F-wave, the value (≈ 4879 MeV) is still higher than that of $D_s^* D_{s2}^*$. Thus, we may safely ignore the possible baryon-antibaryon contributions in this study.

In a 3^{-+} $D^* \bar{D}_2^*$ state, partial waves of S , D , G , and I may all contribute. As a first step exploration, we consider only the dominant S-wave interactions. Possible coupled channel effects will be deferred to future works. The present study is organized as follows. After the introduction in Sec. I, we present the main ingredients for our study in Sec. II. Then we give the numerical results in Sec. III. The final part is for discussions and conclusions.

II. WAVEFUNCTIONS, AMPLITUDES, AND LAGRANGIAN

We study the meson-antimeson bound state problem in a meson exchange model. The potential is derived from the scattering amplitudes [21] and the flavor wave functions of the system are necessary. Since the states we are discussing have a definite C-parity while the combination of $c\bar{q}$ and $\bar{c}q$ mesons does not, a relative sign problem arises between the two parts of a flavor wave function. One has to find the relation between the flavor wave function and the potential with definite C-parity. There are some discussions about this problem in the literatures [21–25]. Here we revisit it by using the G-parity transformation rule which relates the amplitudes between NN and $N\bar{N}$ [26].

Since D mesons do not have defined C-parity, we may assume arbitrary complex phases α and β under the C-parity transformations

$$\begin{aligned} \bar{D}^{*0} &\leftrightarrow \alpha_2 D^{*0}, & D^{*-} &\leftrightarrow \alpha_1 D^{*+}, \\ \bar{D}_2^{*0} &\leftrightarrow \beta_2 D_2^{*0}, & D_2^{*-} &\leftrightarrow \beta_1 D_2^{*+}. \end{aligned} \quad (1)$$

According to the SU(2) transformation, one finds the following isospin doublets

$$\begin{pmatrix} \bar{D}^{*0} \\ D^{*-} \end{pmatrix}, \quad \begin{pmatrix} \alpha_1 D^{*+} \\ -\alpha_2 D^{*0} \end{pmatrix}, \quad \begin{pmatrix} \bar{D}_2^{*0} \\ D_2^{*-} \end{pmatrix}, \quad \begin{pmatrix} \beta_1 D_2^{*+} \\ -\beta_2 D_2^{*0} \end{pmatrix}, \quad (2)$$

from which the G-parity transformations read

$$\begin{aligned} \begin{pmatrix} \bar{D}^{*0} \\ D^{*-} \end{pmatrix} &\rightarrow \begin{pmatrix} \alpha_1 D^{*+} \\ -\alpha_2 D^{*0} \end{pmatrix} \rightarrow \begin{pmatrix} -\bar{D}^{*0} \\ -D^{*-} \end{pmatrix}, \\ \begin{pmatrix} \bar{D}_2^{*0} \\ D_2^{*-} \end{pmatrix} &\rightarrow \begin{pmatrix} \beta_1 D_2^{*+} \\ -\beta_2 D_2^{*0} \end{pmatrix} \rightarrow \begin{pmatrix} -\bar{D}_2^{*0} \\ -D_2^{*-} \end{pmatrix}. \end{aligned} \quad (3)$$

Similar to the study of the $D^* \bar{D}_1$ bound state problem [21], one may construct several states from D^* and \bar{D}_2^* . Here we concentrate only on the $C = +$ case. If the system is an isovector (isoscalar), we label it Z_J (X_J) where J is the angular momentum. Explicitly, one has the G-parity eigenstates

$$\begin{aligned} Z_J^0 &= \frac{1}{2\sqrt{2}} \left[(D^{*-} D_2^{*+} + D_2^{*+} D^{*-}) - \beta_1^\dagger \beta_2 (\bar{D}^{*0} D_2^{*0} + D_2^{*0} \bar{D}^{*0}) \right. \\ &\quad \left. + c\alpha_1 \beta_1^\dagger (D_2^{*-} D^{*+} + D^{*+} D_2^{*-}) - c\alpha_2 \beta_1^\dagger (\bar{D}_2^{*0} D^{*0} + D^{*0} \bar{D}_2^{*0}) \right], \\ X_J^0 &= \frac{1}{2\sqrt{2}} \left[(D^{*-} D_2^{*+} + D_2^{*+} D^{*-}) + \beta_1^\dagger \beta_2 (\bar{D}^{*0} D_2^{*0} + D_2^{*0} \bar{D}^{*0}) \right. \\ &\quad \left. + c\alpha_1 \beta_1^\dagger (D_2^{*-} D^{*+} + D^{*+} D_2^{*-}) + c\alpha_2 \beta_1^\dagger (\bar{D}_2^{*0} D^{*0} + D^{*0} \bar{D}_2^{*0}) \right], \end{aligned}$$

$$+c\alpha_1\beta_1^\dagger(D_2^{*-}D^{*+} + D^{*+}D_2^{*-}) + c\alpha_2\beta_1^\dagger(\bar{D}_2^{*0}D^{*0} + D^{*0}\bar{D}_2^{*0})], \quad (4)$$

where $c = 1$ is the C-parity and the superscript indicates the electric charge. One may check

$$\hat{G}Z_J^0 = -cZ_J^0, \quad \hat{C}X_J^0 = cX_J^0. \quad (5)$$

The procedure to derive the potential is similar to that in Ref. [21]. Now we calculate the amplitude $T(Z_J^0) = \langle Z_J^0 | \hat{T} | Z_J^0 \rangle$ with the G-parity transformation rule (3). We just present several terms to illustrate the derivation. Together with the above Z_J^0 wave function, one has

$$\begin{aligned} T(Z_J^0) &= \frac{1}{4} \left\{ T_{[D_2^{*+} \rightarrow D_2^{*+}, D^{*-} \rightarrow D^{*-}]} - \beta_1\beta_2^\dagger T_{[D_2^{*+} \rightarrow D_2^{*0}, D^{*-} \rightarrow \bar{D}^{*0}]} + c\alpha_1^\dagger\beta_1 T_{[D_2^{*+} \rightarrow D^{*+}, D^{*-} \rightarrow D_2^{*-}]} + \dots \right\} \\ &= \frac{G^\pi}{4} \left\{ T_{[D_2^{*+} \rightarrow D_2^{*+}, D^{*0} \rightarrow D^{*0}]} + \alpha_1^\dagger\alpha_2\beta_1\beta_2^\dagger T_{[D_2^{*+} \rightarrow D_2^{*0}, D^{*0} \rightarrow D^{*+}]} + c\alpha_1^\dagger\alpha_2\beta_1\beta_2^\dagger T_{[D_2^{*+} \rightarrow D^{*+}, D^{*0} \rightarrow D_2^{*0}]} + \dots \right\} \end{aligned} \quad (6)$$

In fact, the convention $\alpha_1\alpha_2^\dagger = \beta_1\beta_2^\dagger$ is implied in the Lagrangian in Eq. (8). So $\alpha_1\alpha_2^\dagger\beta_1\beta_2^\dagger = \alpha_1^\dagger\alpha_2\beta_1\beta_2^\dagger = 1$ and one finally gets

$$\begin{aligned} T(Z_J^0) &= \frac{1}{2}G^\pi \left\{ T_{[D_2^{*+} \rightarrow D_2^{*+}, D^{*0} \rightarrow D^{*0}]} + T_{[D_2^{*+} \rightarrow D_2^{*0}, D^{*0} \rightarrow D^{*+}]} + T_{[D_2^{*0} \rightarrow D_2^{*+}, D^{*+} \rightarrow D^{*0}]} + T_{[D_2^{*0} \rightarrow D_2^{*0}, D^{*+} \rightarrow D^{*+}]} \right. \\ &\quad \left. + cT_{[D_2^{*+} \rightarrow D^{*+}, D^{*0} \rightarrow D_2^{*0}]} + cT_{[D_2^{*+} \rightarrow D^{*0}, D^{*0} \rightarrow D_2^{*+}]} + cT_{[D_2^{*0} \rightarrow D^{*+}, D^{*+} \rightarrow D_2^{*0}]} + cT_{[D_2^{*0} \rightarrow D^{*0}, D^{*+} \rightarrow D_2^{*+}]} \right\}, \\ T(X_J^0) &= \frac{1}{2}G^\pi \left\{ T_{[D_2^{*+} \rightarrow D_2^{*+}, D^{*0} \rightarrow D^{*0}]} - T_{[D_2^{*+} \rightarrow D_2^{*0}, D^{*0} \rightarrow D^{*+}]} - T_{[D_2^{*0} \rightarrow D_2^{*+}, D^{*+} \rightarrow D^{*0}]} + T_{[D_2^{*0} \rightarrow D_2^{*0}, D^{*+} \rightarrow D^{*+}]} \right. \\ &\quad \left. + cT_{[D_2^{*+} \rightarrow D^{*+}, D^{*0} \rightarrow D_2^{*0}]} - cT_{[D_2^{*+} \rightarrow D^{*0}, D^{*0} \rightarrow D_2^{*+}]} - cT_{[D_2^{*0} \rightarrow D^{*+}, D^{*+} \rightarrow D_2^{*0}]} + cT_{[D_2^{*0} \rightarrow D^{*0}, D^{*+} \rightarrow D_2^{*+}]} \right\}. \end{aligned} \quad (7)$$

It is obvious that we may calculate the potential of meson-antimeson interaction from that of meson-meson together with a given Lagrangian for $(c\bar{q})$ meson fields. The arbitrary relative phase in the flavor wave function of a meson-antimeson system is canceled in this procedure. To derive the explicit expression of the potential, one needs interaction Lagrangian.

The Lagrangian for pion interactions in the heavy quark limit and chiral limit reads [27, 28]

$$\begin{aligned} \mathcal{L}_\pi &= g\text{Tr}[H\mathcal{A}\gamma_5\bar{H}] + g''\text{Tr}[T_\mu\mathcal{A}\gamma_5\bar{T}^\mu] \\ &\quad + \left\{ \frac{h_1}{\Lambda_\chi} \text{Tr}[T^\mu(D_\mu\mathcal{A})\gamma_5\bar{H}] + h.c. \right\} + \left\{ \frac{h_2}{\Lambda_\chi} \text{Tr}[T^\mu(\mathcal{D}A_\mu)\gamma_5\bar{H}] + h.c. \right\}, \end{aligned} \quad (8)$$

where

$$\begin{aligned} H &= \frac{1+\not{v}}{2}[P^{*\mu}\gamma_\mu + P\gamma_5], \quad \bar{H} = \gamma^0 H^\dagger \gamma^0 \\ T^\mu &= \frac{1+\not{v}}{2} \left\{ P_2^{*\mu\nu}\gamma_\nu + \sqrt{\frac{3}{2}}P_1^\nu\gamma_5[g_\nu^\mu - \frac{1}{3}\gamma_\nu(\gamma^\mu - v^\mu)] \right\}, \quad \bar{T}^\mu = \gamma^0 T^\dagger \gamma^0. \end{aligned} \quad (9)$$

The fields $P^* = (D^{*0}, D^{*+})$, and $P_2^* = (D_2^{*0}, D_2^{*+})$ annihilate the $c\bar{q}$ mesons. P and P_1 have similar form but we do not involve them in the following calculation. The axial vector field A^μ is defined as $A^\mu = \frac{i}{2}(\xi^\dagger\partial^\mu\xi - \xi\partial^\mu\xi^\dagger)$ with $\xi = \exp(i\mathcal{M}/f)$, $f = 132$ MeV and

$$\mathcal{M} = \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} & \pi^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} \end{pmatrix}. \quad (10)$$

If we further consider σ exchange, one needs additional interaction terms

$$\mathcal{L}_\sigma = g_\sigma\text{Tr}[H\sigma\bar{H}] + g_\sigma''\text{Tr}[T^\mu\sigma\bar{T}_\mu] + \frac{h'_\sigma}{f_\pi}\text{Tr}[T^\mu(\partial_\mu\sigma)\bar{H} + H(\partial_\mu\sigma)\bar{T}^\mu]. \quad (11)$$

The coupling constants must be determined in order for numerical analysis. One extracts the pion coupling constant g from the decay $D^* \rightarrow D\pi$: $g = 0.59 \pm 0.07 \pm 0.01$ [29]. For $h_\chi = \frac{h_1+h_2}{\Lambda_\chi}$, we use the value 0.55 GeV^{-1} estimated in Ref. [27]. To determine the coupling constant g'' , we turn to the chiral quark model [30] with which one may get the relation $g'' = -g$.

For the sigma coupling constants, we can just get estimates from the chiral quark model or the chiral multiplet assumption [31]. These approaches have been used in the baryon case [32] for the purpose of cross checking, where we get consistent results. Now the former method may give the relation $g''_\sigma = -g_\sigma$ and the value $g_\sigma = g_{ch} = 2.621$ if one adopts the Lagrangian [30]

$$L_I = -g_{ch}\bar{\psi}(\sigma + i\gamma_5\pi_a\tau_a)\psi, \quad (12)$$

where $\psi = (u, d)^T$ is the quark field and τ_a the Pauli matrix. One should note the normalization problem in this approach [28, 33]. However, if one estimates g_σ from the chiral multiple assumption, a value less than 1 is obtained. For the remaining h'_σ , no available approach may be used. Since the large uncertainties of the coupling constants, we will select several values to see the σ -exchange effects on the conclusions.

In deriving the above relations for the coupling constants, we have used the polarization vectors $\varepsilon_{\pm 1}^\mu = \frac{1}{\sqrt{2}}(0, \pm 1, i, 0)$ and $\varepsilon_0^\mu = (0, 0, 0, -1)$ for the vector meson D^* and

$$\begin{aligned} \varepsilon_{\pm 2}^{\mu\nu} &= \varepsilon_{\pm 1}^\mu \varepsilon_{\pm 1}^\nu, \\ \varepsilon_{\pm 1}^{\mu\nu} &= \sqrt{\frac{1}{2}}[\varepsilon_{\pm 1}^\mu \varepsilon_0^\nu + \varepsilon_0^\mu \varepsilon_{\pm 1}^\nu], \\ \varepsilon_0^{\mu\nu} &= \sqrt{\frac{1}{6}}[\varepsilon_{+1}^\mu \varepsilon_{-1}^\nu + \varepsilon_{-1}^\mu \varepsilon_{+1}^\nu + 2\varepsilon_0^\mu \varepsilon_0^\nu], \end{aligned} \quad (13)$$

for the tensor meson D_2^* [34].

III. POTENTIALS AND NUMERICAL ANALYSIS

Now one may derive the potentials through the amplitudes in (7). Using the same procedure as Ref. [21], one gets the one-pion-exchange potential (OPEP) for S-wave interaction in the case $I = 1$

$$V^\pi(Z_J) = -\frac{gg''}{6f^2}G^\pi C_d \left[\delta(\vec{r}) - \frac{m_\pi^2 e^{-m_\pi r}}{4\pi r} \right] + \frac{|h_\chi|^2}{15f^2}cG^\pi C_e \left[\nabla^2 \delta(\vec{r}) - \mu^2 \delta(\vec{r}) - \frac{\mu^4}{4\pi r} \cos(\mu r) \right], \quad (14)$$

where $\mu = \sqrt{(m_{D_2} - m_{D^*})^2 - m_\pi^2}$, and

$$C_d = \begin{cases} -1, & (J=3) \\ \frac{1}{2}, & (J=2) \\ \frac{3}{2}, & (J=1) \end{cases}, \quad C_e = \begin{cases} \frac{1}{2}, & (J=3) \\ -\frac{1}{2}, & (J=2) \\ -\frac{1}{4}, & (J=1) \end{cases}. \quad (15)$$

There are two parts in the potential: direct part and spin-exchange part. The later corresponds to the terms containing c in Eq. (7). For the case of $I = 0$, $V^\pi(X_J) = -3V^\pi(Z_J)$.

The singular behavior at small distances needs to be regularized [35]. If a form factor $FF = \left(\frac{\Lambda^2 - m^2}{\Lambda^2 - q^2} \right)^2$ is added to each vertex, one finally has

$$\begin{aligned} V^\pi(Z_J) &= -\frac{gg''}{6f^2}G^\pi C_d \left[-\frac{m_\pi^2}{4\pi r}(e^{-m_\pi r} - e^{-\Lambda r}) + \frac{m_\pi^2 \eta^2}{8\pi \Lambda} e^{-\Lambda r} \right. \\ &\quad + \frac{m_\pi^2 \eta^4}{32\pi \Lambda^3} (1 + \Lambda r) e^{-\Lambda r} + \frac{\eta^6}{192\pi \Lambda^3} (3 + 3\Lambda r + \Lambda^2 r^2) e^{-\Lambda r} \Big] \\ &\quad + \frac{|h_\chi|^2}{15f^2}cG^\pi C_e \left\{ -\frac{\mu^4}{4\pi r} (\cos(\mu r) - e^{-\alpha r}) + \frac{\mu^4 \eta^2}{8\pi \alpha} e^{-\alpha r} \right. \\ &\quad \left. - \frac{\mu^2 \eta^4}{32\pi \alpha} (1 + \alpha r) e^{-\alpha r} - \frac{\eta^6}{192\pi \alpha} (3 + 3\alpha r - \alpha^2 r^2) e^{-\alpha r} \right\}, \end{aligned} \quad (16)$$

where $\eta = \sqrt{\Lambda^2 - m_\pi^2}$, and $\alpha = \sqrt{\Lambda^2 - (m_{D_2} - m_{D^*})^2}$.

Similarly, the one- σ -exchange potential (OsEP) is

$$V^\sigma(Z_J) = g_\sigma g''_\sigma \left[\frac{1}{4\pi r} (e^{-m_\sigma r} - e^{-\Lambda r}) - \frac{\eta_\sigma^2}{8\pi \Lambda} e^{-\Lambda r} - \frac{\eta_\sigma^4}{32\pi \Lambda^3} (1 + \Lambda r) e^{-\Lambda r} \right]$$

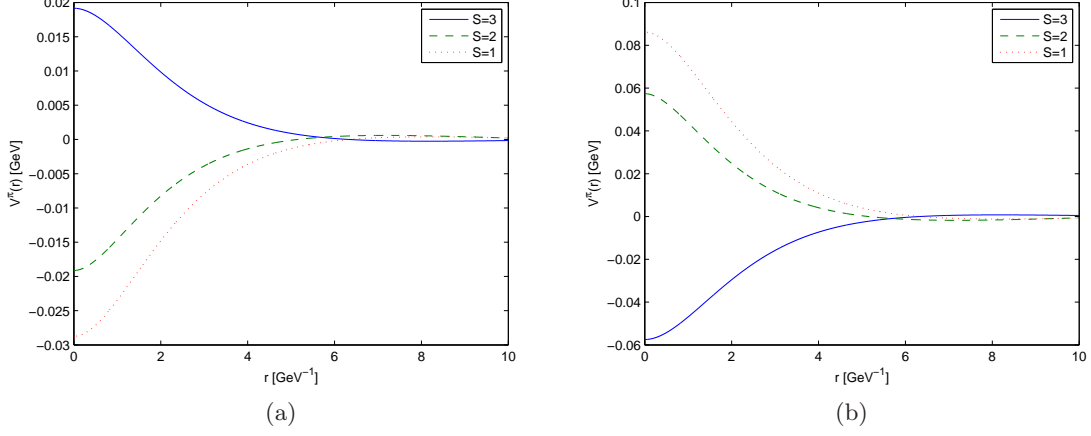


FIG. 2: One-pion-exchange potentials for (a) Z states and (b) X states with the cutoff $\Lambda = 1$ GeV.

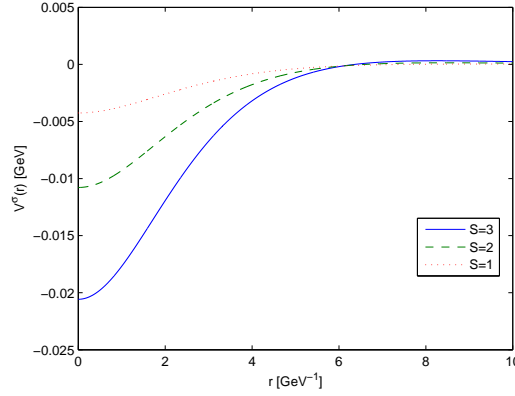


FIG. 3: One-sigma-exchange potentials for X and Z states with the cutoff $\Lambda = 1$ GeV and the coupling constants $g''_\sigma = -g_\sigma = -1.0$, $h'_\sigma = 1.0$.

$$\begin{aligned}
 & -\frac{\eta_\sigma^6}{192\pi\Lambda^5}(3+3\Lambda r+\Lambda^2 r^2)e^{-\Lambda r}] \\
 & +\frac{|h'_\sigma|^2}{3f_\pi^2}C_\sigma\left[\frac{\mu_\sigma^2}{4\pi r}(e^{-\mu_\sigma r}-e^{-\alpha r})-\frac{\mu_\sigma^2\eta_\sigma^2}{8\pi\alpha}e^{-\alpha r}-\frac{\mu_\sigma^2\eta_\sigma^4}{32\pi\alpha^3}(1+\alpha r)e^{-\alpha r}\right. \\
 & \left.-\frac{\eta_\sigma^6}{192\pi\alpha^3}(3+3\alpha r+\alpha^2 r^2)e^{-\alpha r}\right], \\
 V^\sigma(X_J) &= V^\sigma(Z_J),
 \end{aligned} \tag{17}$$

where $\mu_\sigma = \sqrt{m_\sigma^2 - (m_{D_2} - m_{D^*})^2}$, $\eta_\sigma = \sqrt{\Lambda^2 - m_\sigma^2}$. The coefficient $C_\sigma = 1$ for $J = 3$, $\frac{1}{2}$ for $J = 2$, and $\frac{1}{6}$ for $J = 1$. The spin-dependent nature of OsEP comes from the third coupling term in the Lagrangian (11).

Before the numerical calculation, we take a look at the relative strengthes of potentials. For the meson masses, we use $m_\pi = 137.27$ MeV, $m_{D^*} = 2008.63$ MeV, and $m_{D_2} = 2463.5$ MeV [20]. We plot OPEPs with $\Lambda = 1$ GeV in Fig. 2. It is obvious that X_3 is the most attractive case. For the σ meson exchange contribution, we use $m_\sigma = 600$ MeV for the illustration. In Fig. 3, we show OsEPs with $g''_\sigma = -g_\sigma = -1.0$, $h'_\sigma = 1.0$, and $\Lambda = 1$ GeV. It is interesting that the potential for X_3 is also the most attractive one. Thus the long-range and medium-range meson-exchanges are both helpful for the formation of a $I^G(J^{PC}) = 0^-(3^{-+})$ state.

Now we turn to the numerical results for the OPEP case by solving the Schrödinger equation. In the potential, there is an unknown phenomenological cutoff parameter Λ . It incorporates the size information of the interacting mesons. If Λ goes to infinity, the potential describes the interactions of structureless mesons. A small cutoff is relevant to the real case. In principle, an appropriate value should be around 1 GeV which is realized from the nuclear potential models. Since we do not want to give accurate prediction on the binding energy, this parameter is not fixed. We

just tune its value in a range and check whether a bound state exists or not. The results of binding energy (B.E.) and root-mean-square radius (r_{rms}) for X_3 with various Λ are presented in Tab. I, where the cases for $r_{rms} < 0.8$ fm or $\Lambda > 4$ GeV are neglected. Similarly, one may get numerical results for other possibilities, which are also given in that table. The resultant cutoff much larger than 1 GeV indicates that the attraction is not strong enough for the formation of a hadronic bound state. Comparing the three cases in the table, of course X_3 is more likely to be existent.

State	Λ (GeV)	B.E. (MeV)	r_{rms} (fm)
X_3	2.3	0.6	3.8
	2.4	3.7	1.6
	2.5	9.9	1.0
	2.6	19.8	0.8
Z_2	3.8	2.5	1.9
Z_1	3.6	1.9	2.2
	3.7	8.4	1.1

TABLE I: Cutoff (Λ), binding energy (B.E.) and root-mean-square radius (r_{rms}) for X and Z states with OPEP. We do not show results for $\Lambda > 4$ GeV or $r_{rms} < 0.8$ fm.

The minimal cutoff for a binding solution is a little larger than 2 GeV if we consider only π -exchange. This means that additional attraction may lower the value to a more appropriate number. We would like to check how much attraction the sigma meson contributes. Because of the large uncertainty of the coupling constants, we take three sets of them: (1) $g_\sigma = 2.621$, $g''_\sigma = -g_\sigma$, $h'_\sigma = 0$, which corresponds to neglecting spin-exchange potential; (2) $g_\sigma = 1.0$, $g''_\sigma = -g_\sigma$, $h'_\sigma = 1$; and (3) $g_\sigma = 2.621$, $g''_\sigma = -g_\sigma$, $h'_\sigma = 2.621$. The last set is the most attractive case. After the solution of the Schrödinger equation, the cutoff parameters satisfying the condition $r_{rms} > 0.8$ fm and $\Lambda < 4$ GeV are summarized in Tab. II. From the resultant cutoff parameters, we see that the existence of X_3 is probable. Its mass should be around the $D^*D_2^*$ threshold (≈ 4472 MeV).

States	Set 1	Set 2	Set 3
X_3	1.7~2.2	1.5~1.7	1.0~1.1
X_2			1.4~1.5
X_1			
Z_3		2.7~3.0	1.1~1.2
Z_2	2.6~3.2	2.5~2.7	1.2~1.3
Z_1	2.2~2.9	2.8~3.1	1.5~1.7

TABLE II: Cutoff values (GeV) for X and Z states with OPEP+OsEP when binding solutions exist. We do not show cutoffs if $\Lambda > 4$ GeV or $r_{rms} < 0.8$ fm.

IV. DISCUSSIONS AND CONCLUSIONS

From the meson exchange potentials and the numerical analysis, one has found that the most probable molecule in the $C = + D^* \bar{D}_2^*$ system is X_3 . If the state really exists, it may decay through its components, i.e. $D^* \rightarrow D\pi$, $D_2^* \rightarrow D\pi$, or $D_2^* \rightarrow D^*\pi$. The X_3 may also decay through the quark rearrangement, i.e. the final states are a $c\bar{c}$ meson and a $q\bar{q}$ ($q = u, d$) meson. The later type decay may be used to identify the exotic quantum numbers. Here we focus only on this case.

For convenience of discussion, we assume that L is the relative orbital momentum between the $c\bar{c}$ and the $q\bar{q}$ mesons and relax the isospin requirement temporarily. Since the spins of the charm quark and the light quark in both D^* and D_2^* are parallel, the spin of $c\bar{c}$ in X_3 must be 1. According to the heavy quark spin symmetry, the spin of the final charmonium after rearrangement should also be $S = 1$. Thus the final $c\bar{c}$ state can only be ψ or χ_{cJ} . The decay channels are obtained as follows:

- (1) If the final $c\bar{c}$ is J/ψ , the J^{PC} of the produced $q\bar{q}$ meson may be $(1 \sim 5)^{--}$ for $L = 1$, $(1, 3, 5)^{+-}$ for $L = 2$, $(1 \sim 7)^{--}$ for $L = 3$, and so on. After some inspections on the meson masses, one finds that kinematically allowed decays for the X_3 molecule are just $J/\psi\rho$ and $J/\psi\omega$ with $L = 1, 3, 5$, and $J/\psi h_1(1170)$ and $J/\psi b_1(1235)$ with $L = 2, 4$. If it is $\psi(2S)$, the kinematically allowed decays are $\psi(2S)\rho$ and $\psi(2S)\omega$ with $L = 1, 3, 5$.

(2) If the $c\bar{c}$ is χ_{c0} , the J^{PC} of the $q\bar{q}$ meson may be $(2 \sim 4)^{++}$ for $L = 1$, $(2, 4)^{-+}$ for $L = 2$, $(0 \sim 6)^{++}$ for $L = 3$, and so on. The kinematically allowed decays are $\chi_{c0}f_0(500)$, $\chi_{c0}f_0(980)$, and $\chi_{c0}a_0(980)$ with $L = 3$.

(3) If the $c\bar{c}$ is χ_{c1} , the J^{PC} of the $q\bar{q}$ meson may be $(2, 4)^{-+}$ for $L = 0$, $(1 \sim 5)^{++}$ for $L = 1$, $(0, 2, 4, 6)^{-+}$ for $L = 2$, $(0 \sim 7)^{++}$ for $L = 3$, and so on. The kinematically allowed decays are $\chi_{c1}\pi$, $\chi_{c1}\eta$, $\chi_{c1}\eta'$ with $L = 2, 4$, and $\chi_{c1}f_0(500)$ with $L = 3$.

(4) If the $c\bar{c}$ is χ_{c2} , the J^{PC} of the $q\bar{q}$ meson may be $(2, 4)^{-+}$ for $L = 0$, $(0 \sim 6)^{++}$ for $L = 1$, $(0, 2, 4, 6)^{-+}$ for $L = 2$, $(0 \sim 8)^{++}$ for $L = 3$, and so on. The kinematically allowed decays are $\chi_{c2}f_0(500)$ with $L = 1, 3, 5$, and $\chi_{c2}\pi$ and $\chi_{c2}\eta$ with $L = 2, 4$.

Therefore, the allowed two-body strong decays for X_3 are $J/\psi\omega$ (PFH), $\psi(2S)\omega$ (PFH), $J/\psi h_1(1170)$ (DG), $\chi_{c0}f_0(500)$ (F), $\chi_{c0}f_0(980)$ (F), $\chi_{c1}\eta$ (DG), $\chi_{c1}\eta'$ (DG), $\chi_{c1}f_0(500)$ (F), $\chi_{c2}f_0(500)$ (PFH), and $\chi_{c2}\eta$ (DG). There is no S-wave decay. Because high L processes are suppressed and $\psi(2S)$ and χ_{c2} are excited states, the simplest way to identify X_3 may be through the $J/\psi\omega$ channel.

Let us analyze the J^{PC} of an assumed state $X(4472)$ observed in the $J/\psi\omega$ mass distribution. Since J/ψ and ω are both $J^{PC} = 1^{--}$ mesons, the quantum numbers of $J/\psi\omega$ are $(0, 1, 2)^{++}$ for S-wave combination, $(0 \sim 3)^{-+}$ for P-wave combination, $(0 \sim 4)^{++}$ for D-wave combination, and so on. What we are interested in is the case that the partial wave is determined to be P . If X were a conventional $c\bar{c}$ meson, the state is $\eta_c(4472)$ and the spin of $c\bar{c}$ must be 0. Because of the heavy quark spin symmetry, the decay $\eta_c(4472) \rightarrow J/\psi\omega$ is suppressed. Then $X(4472)$ could be a hadronic state. Although other meson-antimeson pairs may also form molecules with $J^{PC} = (0 \sim 2)^{-+}$, the masses are smaller. Therefore, based on our numerical analysis, this X around the $D^*\bar{D}_2^*$ threshold is very likely to be a state with the exotic $J^{PC} = 3^{-+}$.

If one wants to look for Z_3 , one can use those kinematically allowed decay channels, $J/\psi\rho$ (PFH), $\psi(2S)\rho$ (PFH), $J/\psi b_1(1235)$ (DG), $\chi_{c0}a_0(980)$ (F), $\chi_{c1}\pi$ (DG), and $\chi_{c2}\pi$ (DG). The practical way to identify the J^{PC} is to analyze the partial wave of $J/\psi\rho$. The search is also helpful to test the meson exchange models.

We consider only S-wave interactions of D^* and \bar{D}_2^* in this paper. Higher partial waves also have contributions to the 3^{-+} molecular bound state. Additional attraction from the channel coupling may reinforce the present preliminary result. Future studies on such effects and production cross section may be helpful for the search on exotic states.

Replacing a c quark with a b quark, one may study the bottom case. Because the production of a hidden bottom molecule $B^*\bar{B}_2^*$ needs much higher energy and the production cross section is smaller, it is difficult for experimentalists to explore this case in near future. However, with the replacement $c \rightarrow s$, one may study whether there is a bound state or resonance with $J^{PC} = 3^{-+}$ near the $K^*K_2(\approx 2322 \text{ MeV})$ threshold. If such a state exists, it may decay into $\omega\phi$ and can be detected.

In short summary, we have investigated whether hadronic bound states exist in the $D^*\bar{D}_2^*$ system in a one-boson-exchange model. The $C = +$ case is discussed in this paper. We find that the $I^G(J^{PC}) = 0^+(3^{-+})$ X_3 state is the most probable one. A feasible place to identify it may be in the invariant mass distribution of $J/\psi\omega$ around 4472 MeV. A similar study for a state around 2322 MeV in the $\omega\phi$ mass distribution is also called for.

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